

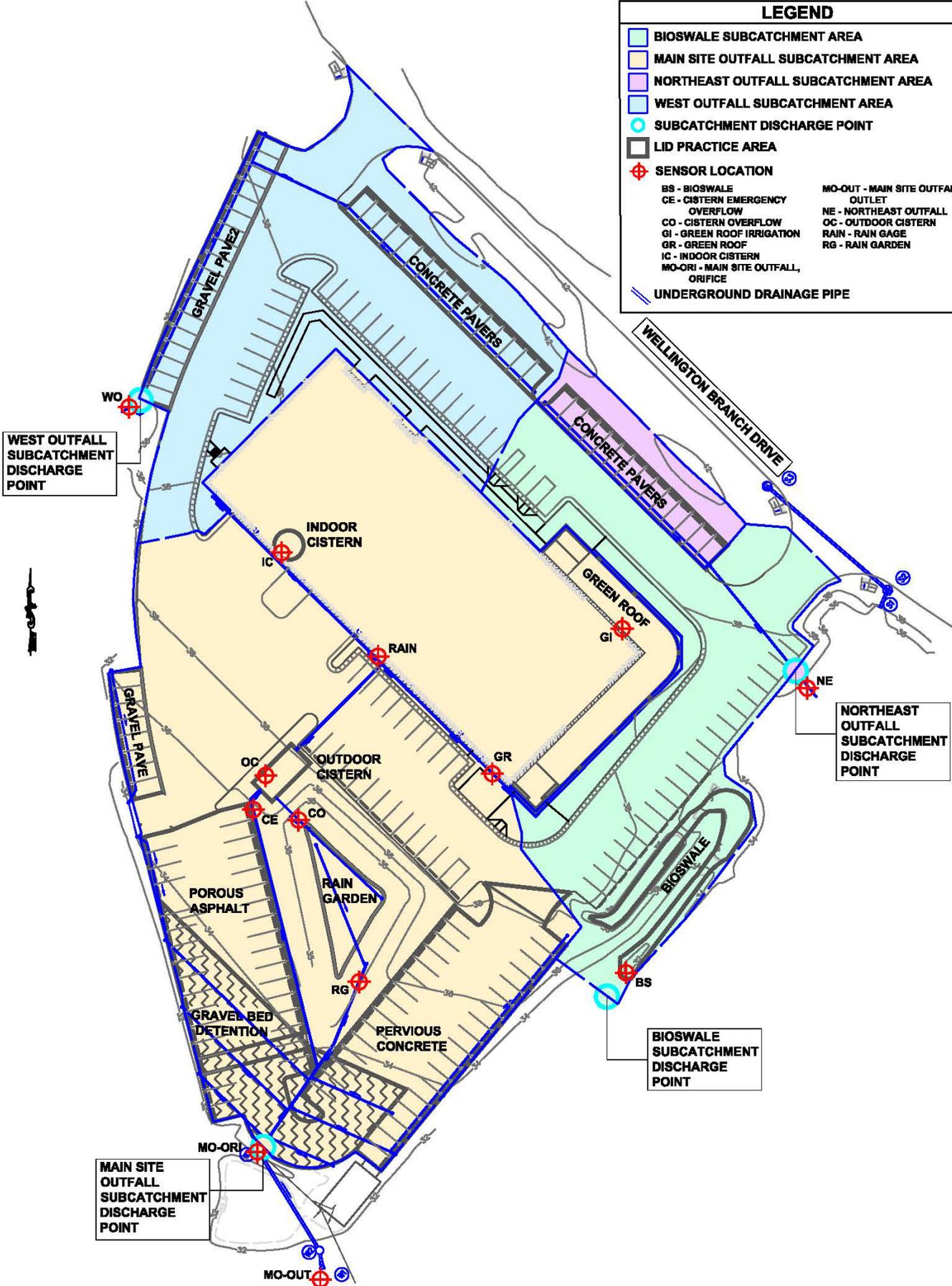


# WSSI LID ANALYSIS WEBSITE: SAMPLE CALCULATIONS



### LEGEND

<span style="display: inline-block; width: 15px; height: 10px; background-color: #ADD8E6; border: 1px solid black;"></span> BIOSWALE SUBCATCHMENT AREA	
<span style="display: inline-block; width: 15px; height: 10px; background-color: #FFD700; border: 1px solid black;"></span> MAIN SITE OUTFALL SUBCATCHMENT AREA	
<span style="display: inline-block; width: 15px; height: 10px; background-color: #DDA0DD; border: 1px solid black;"></span> NORTHEAST OUTFALL SUBCATCHMENT AREA	
<span style="display: inline-block; width: 15px; height: 10px; background-color: #ADD8E6; border: 1px solid black;"></span> WEST OUTFALL SUBCATCHMENT AREA	
<span style="display: inline-block; width: 10px; height: 10px; border: 1px solid black; border-radius: 50%;"></span> SUBCATCHMENT DISCHARGE POINT	
<span style="display: inline-block; width: 15px; height: 10px; border: 1px solid black;"></span> LID PRACTICE AREA	
<span style="display: inline-block; width: 10px; height: 10px; border: 1px solid black; border-radius: 50%;"></span> SENSOR LOCATION	
BS - BIOSWALE	MO-OUT - MAIN SITE OUTFALL, OUTLET
CE - CISTERN EMERGENCY OVERFLOW	NE - NORTHEAST OUTFALL
CO - CISTERN OVERFLOW	OC - OUTDOOR CISTERN
GI - GREEN ROOF IRRIGATION	RAIN - RAIN GAGE
GR - GREEN ROOF	RG - RAIN GARDEN
IC - INDOOR CISTERN	
MO-ORI - MAIN SITE OUTFALL, ORIFICE	
<span style="display: inline-block; width: 15px; border-bottom: 1px solid black;"></span> UNDERGROUND DRAINAGE PIPE	



<b>Nomenclature</b>			$t_L$	hr	lag time (time to peak)
$A$	ac	subcatchment area	$V$	ft/s	flow velocity
$a$	ft <sup>2</sup>	flow area	$x$	ft <sup>1</sup>	sensor-read value
$C$	-	Rational Method coefficient	$y$	ft	distance
$C_d$	-	coefficient of discharge (orifice)	$z$	ft	zero-flow depth
$C_w$	-	weir coefficient			
$CN$	-	curve number	$\alpha_1$	-	coefficient for quadratic equation
$c$	ft	circular segment height	$\alpha_2$	-	coefficient for quadratic equation
$c_{gr}$	ft	green roof calibration coefficient	$\alpha_3$	-	coefficient for quadratic equation
$D$	ft	diameter	$\delta$	ft/ft	effective pipe slope
$d$	ft	flow depth	$\Phi$	rad	central angle
$g$	ft/s <sup>2</sup>	gravitational acceleration	$\theta$	deg	weir notch angle
$H$	ft	weir depth			
$h$	ft	head			
$l$	ft	underdrain invert depth	$BS$		Bioswale monitoring station
$l_a$	in	initial abstractions	$CE$		Cistern Emergency Overflow monitoring station
$i$	in/hr	rainfall intensity	$GR$		Green Roof monitoring station
$K$	ft <sup>2</sup>	circular segment area	$MO-ORI$		Main Site Outfall monitoring station, orifice sensor
$n$	-	Manning's roughness coefficient	$MO-OUT$		Main Site Outfall monitoring station, outlet sensor
$P$	in	precipitation depth	$NE$		Northeast Outfall monitoring station
$P_w$	ft	wetted perimeter	$O$		Orifice
$Q$	gal	runoff volume	$p$		peak
$q$	cfs, gpm	flow rate	$RG$		Rain Garden monitoring station
$R$	ft	radius	$WO$		West Outfall Monitoring station
$R_h$	ft	hydraulic radius	$O$		zero-flow
$r$	-	runoff retention			
$S$	in	potential retention			
$s$	ft	arc length			
$t_c$	hr	time of concentration			

### Symbols

### Subscripts

## Rainfall Calculations

Rainfall data is provided as incremental depths recorded every 6 minutes. The following are sample calculations necessary to process rainfall data for analysis. Sample calculations are based on the sample rainfall data in Table 1.

**Table 1.** Sample rainfall data.

Time	Incremental Depth (in)
3:30	0.00
3:36	0.02
3:42	0.10
3:48	0.12
3:54	0.04
4:00	0.01
4:06	0.00

<sup>1</sup> All pressure transducers report depth values in inches, but the depths are converted to feet for all calculations. Therefore, for purposes of this report it will be assumed that the units for sensor readings are always feet.

## - Total Rainfall

The total rainfall from a storm event is the sum of all incremental rainfall depths. For the data in Table 1, the total rainfall is given by:

$$P = 0.00 \text{ in} + 0.02 \text{ in} + 0.10 \text{ in} + 0.12 \text{ in} + 0.04 \text{ in} + 0.01 \text{ in} + 0.00 \text{ in} = 0.29 \text{ in}$$

## - Rainfall Intensity

To calculate rainfall intensity in in/hr, each 6-minute rainfall increment is multiplied by 10, as shown in the calculation below:

$$\frac{\blacksquare \text{ in}}{6 \text{ min}} * \frac{60 \text{ min}}{1 \text{ hr}} = 10 * \blacksquare \frac{\text{in}}{\text{hr}}$$

For example, the incremental rainfall at 3:54 of the sample data is 0.04 in. The rainfall intensity at that same time is:

$$i = 10 * 0.04 \frac{\text{in}}{\text{hr}} = 0.4 \frac{\text{in}}{\text{hr}}$$

## - Time of Rainfall Centroid

To locate the rainfall centroid, total rainfall must first be calculated. The rainfall centroid occurs at the point where cumulative rainfall equals half of the ultimate rainfall total. For example, the total rainfall in the sample storm is 0.29 in (provided above). Therefore, the rainfall centroid occurs when rainfall totals reach 0.145 in, or sometime between 3:42 and 3:48.

# Flow Rate Calculations

## - Manning's Equation: Underdrain Network (CE, WO, RG, NE)

All of the sensors in this group are pressure transducers which read the depth of water in a stilling well assembly as shown in Figure 1. The depths are then converted to flow rates using Manning's equation. The system parameters necessary to run these calculations are described below.

Each monitoring station measures flow conveyed in an underdrain pipe; pipe radii<sup>2</sup> are given below.

$$R_{CE} = 3.99 \text{ in} = 0.333 \text{ ft}$$

$$R_{WO} = 3.03 \text{ in} = 0.253 \text{ ft}$$

$$R_{RG} = 3.03 \text{ in} = 0.253 \text{ ft}$$

$$R_{NE} = 2.01 \text{ in} = 0.168 \text{ ft}$$

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<sup>2</sup> Source: Pacific Plastics. (2012) "IPS Solvent Weld ASTM D1785/D2665 White Sch40". <<http://www.pacificplasticsinc.com/products/solvent-weld/ips-sch40>> (Apr 30, 2012).

All the pipes at monitoring stations are made from PVC, so they all have the same value<sup>3</sup> for Manning's  $n$ .

$$n = 0.009$$

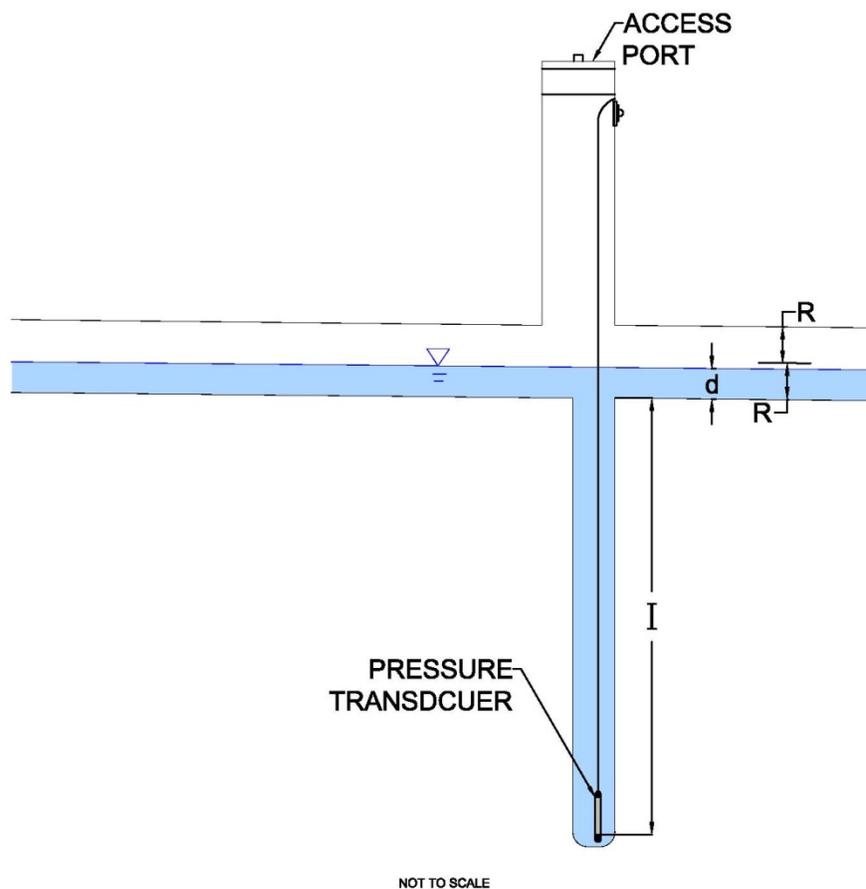
The slope of the pipes conveying flow was the only unknown variable in the Manning's flow equation. These effective pipe slopes<sup>4</sup> were calibrated using data collected in the field (see Appendix B).

$$\delta_{CE} = 0.032 \text{ ft/ft}$$

$$\delta_{WO} = 0.004 \text{ ft/ft}$$

$$\delta_{RG} = 0.018 \text{ ft/ft}$$

$$\delta_{NE} = 0.0045 \text{ ft/ft}$$



**Figure 1.** Diagram of typical stilling well assembly.

<sup>3</sup> Source: Parmley, R. O. (1992). *Hydraulics Field Manual*. McGraw-Hill, Inc., New York, NY, 299.

Lindeburg, M.R. (2008). *Civil engineering reference manual for the PE exam*. Professional Publications, Inc., Belmont, CA, A-36

<sup>4</sup> Pipe slopes are termed "effective" because they were calculated using Manning's equation. Measurement of the actual slopes was not possible because pipes are buried several feet underground.

The zero-flow depth is the maximum water level above a sensor that can be reached before flow occurs at that sensor. For these monitoring stations, the zero-flow depth is the distance from the sensor to the invert of the underdrain or leader pipe. It is assumed that whenever runoff is not occurring, the water level in the stilling well is equal to the zero-flow depth. Therefore, the zero-flow depths for each storm event are calculated based on an average of the last ten sensor readings prior to the beginning of rainfall.

For the sample data shown in Table 2, the storm begins at 1:24. In this case, the values from 1:14 to 1:23 would be averaged together to find the zero-flow depth that would be used for this storm.

$$z_{NE} = \bar{x}_{NE} = \frac{1.4690 \text{ ft} + 1.4696 \text{ ft} + \dots + 1.4695 \text{ ft}}{10} = 1.469 \text{ ft}$$

**Table 2.** Sample Northeast Outfall sensor data.

Time	Sensor Reading (ft)
1:14	1.4690
1:15	1.4696
1:16	1.4691
1:17	1.4685
1:18	1.4686
1:19	1.4694
1:20	1.4696
1:21	1.4690
1:22	1.4685
1:23	1.4695
1:24	1.4693

The first step in calculating flow rate at a stilling well is to determine whether flow is occurring. If the depth of water indicated by the pressure transducer is equal to or less than the zero-flow depth, then there is no water flowing through the pipe. If the water level is above the zero-flow depth, then there is water in the pipe and flow is occurring. For example, at the northeast outfall the zero-flow depth is:

$$z_{NE} = 1.469 \text{ ft}$$

If the sensor records a depth of 1.459 ft, then:

$$x_{NE} = 1.459 \text{ ft} < z_{NE}$$

And it is assumed that no flow is occurring. If the sensor records a depth of 1.489 ft, then:

$$x_{NE} = 1.489 \text{ ft} > z_{NE}$$

$$d = x_{NE} - z_{NE} = 0.02 \text{ ft}$$

In this case, flow is assumed to be occurring and the flow depth is 0.02 ft.

Once it has been determined that flow is occurring the flow rate can be determined using Manning's equation. All underdrains use circular pipes, and if the pipes are not full then the open channel flow assumption necessary for use of Manning's equation is valid. Before applying Manning's equation the hydraulic radius must be calculated. Hydraulic radius is found using one of the methods shown in Table 2<sup>5</sup>. Selection of the method is based on whether the depth of flow is greater than or less than the pipe radius.

For example, for a flow depth of 0.02 ft at the northeast outfall sensor:

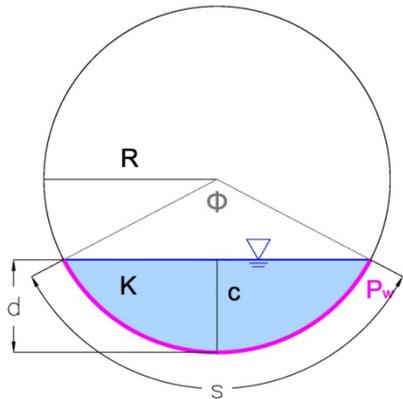
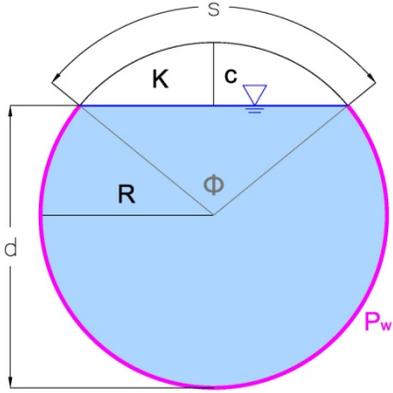
$$d = 0.02 \text{ ft} < R_{NE}$$

$$c = d = 0.02 \text{ ft}$$

$$\varphi = 2 * \arccos\left(\frac{R_{NE} - c}{R_{NE}}\right) = 0.99 \text{ rad}$$

$$K = \frac{R_{NE}^2 * (\varphi - \sin(\varphi))}{2} = 0.0021 \text{ ft}^2$$

**Table 3.** Steps to calculate a hydraulic radius for open channel flow in pipes.

Step	Solving For	Flow depth (d) < Radius (R)	Flow depth (d) > Radius (R)
			
1	circular segment height	$c = d$	$c = 2R - d$
2	central angle	$\varphi = 2\arccos\left(\frac{R - c}{R}\right)$	$\varphi = 2\arccos\left(\frac{R - c}{R}\right)$
3	circular segment area	$K = \frac{R^2(\varphi - \sin\varphi)}{2}$	$K = \frac{R^2(\varphi - \sin\varphi)}{2}$
4	arc length	$s = R\varphi$	$s = R\varphi$
5	flow area	$a = K$	$a = \pi R^2 - K$
6	wetted perimeter	$P_w = s$	$P_w = 2\pi R - s$
7	hydraulic radius	$R_h = \frac{a}{P_w}$	$R_h = \frac{a}{P_w}$

<sup>5</sup> Source: AJ Design Software. (2011). "Hydraulic Radius Design Equations Formulas Calculator." <[http://www.ajdesigner.com/phphydraulicradius/hydraulic\\_radius\\_equation\\_pipe.php](http://www.ajdesigner.com/phphydraulicradius/hydraulic_radius_equation_pipe.php)> (May 2, 2012).

$$s = R_{NE} * \varphi = 0.16 \text{ ft}$$

$$a = K = 0.0021 \text{ ft}^2$$

$$P_w = s = 0.16 \text{ ft}$$

$$R_h = \frac{a}{P_w} = 0.013 \text{ ft}$$

Once the hydraulic radius has been solved for, Manning's equation can be used. As mentioned in the previous section, the Manning's coefficient is the same for all the pipes where sensors are located. The effective slope,  $\delta$ , is specific to each stilling well assembly.

Continuing with the sample calculation for the northeast outfall,

$$V = \frac{1.49}{n} * R_h^{2/3} * \delta_{NE}^{1/2} = 0.62 \text{ ft/s}$$

This gives the velocity of flow; to calculate a flow rate the velocity must be multiplied by the flow area, which was calculated in step 5 of the process shown in Table 3.

$$q = V * a = 0.0013 \text{ cfs} = 0.58 \text{ gpm}$$

### - Manning's Equation: Green Roof Leader Pipe

The green roof flow calculation also uses Manning's equation, and so it requires the same parameters (pipe radius, Manning's n, effective slope, and zero-flow depth) as the underdrain network calculations shown in the previous section.

$$R_{GR} = 3.03 \text{ in} = 0.253 \text{ ft}$$

$$n = 0.009$$

$$\delta_{GR} = 0.0002 \text{ ft/ft}$$

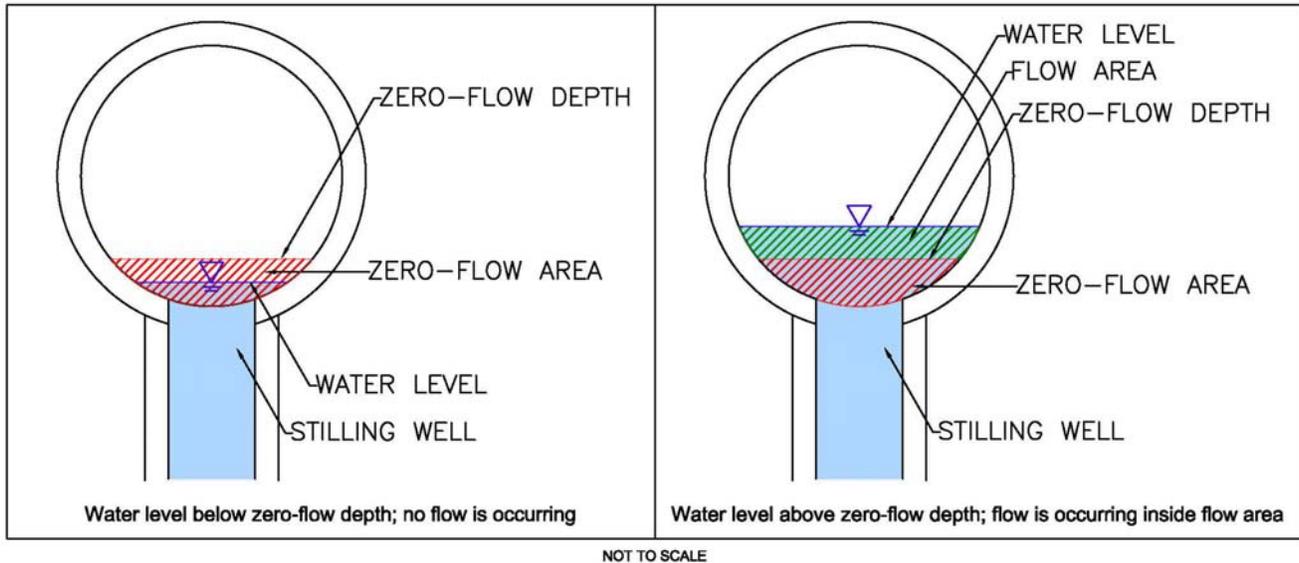
$$z_{GR} = I_{GR} + 0.10 \text{ ft}^6$$

The basic procedure used for green roof flow calculation is the same as the other monitoring points in the underdrain network – depth of flow is determined and Manning's equation applied. However, this station presents an additional challenge in that the pipe permanently has stagnant water in the bottom, meaning that the zero-flow depth is located above the invert of the pipe and there is a corresponding "zero-flow area" between the zero-flow depth and the perimeter of the pipe (Figure 2). If the water level is below the zero-flow depth, there is

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<sup>6</sup> The green roof monitoring point is the only one at which the pipe invert and zero-flow depth are not equal. The sensor values corresponding to green roof leader pipe invert and zero-flow depth are also unique in that they are not constant and must be calculated on a per-storm basis, as described in the "Flow Rate Calculations" section.

no flow occurring in the pipe. If the water level is above the zero-flow depth, the flow area is equal to the total area between the water level and pipe perimeter minus the zero-flow area.



**Figure 2.** Diagram showing the zero-flow area in the green roof leader pipe.

As described in the previous section, the zero-flow depth for each storm event is calculated based on an average of the last ten sensor readings prior to the beginning of rainfall. For the sample data shown in

Table 4 below, rainfall begins at 7:54, so the sensor readings from 7:44 to 7:53 would be averaged together.

$$\overline{x_{GR}} = \frac{1.3639 \text{ ft} + 1.3639 \text{ ft} + \dots + 1.3624 \text{ ft}}{10} = 1.363 \text{ ft}$$

This average sensor reading is related to the zero-flow depth by a calibration coefficient,  $c_{GR}$ . The procedure to determine the coefficient is described in Appendix B, and the coefficient was then verified using data from multiple storm events.

$$c_{GR} = 0.021 \text{ ft}$$

Using this calibration coefficient on the average sensor reading found above gives the following zero-flow depth.

$$z_{GR} = \overline{x_{GR}} + c_{GR} = 1.384 \text{ ft}$$

As mentioned before, the green roof monitoring point is unique in that the zero-flow depth and pipe invert are at different elevations due to permanent stagnant water in the bottom of the pipe. The distance from the zero-flow depth to the pipe invert has been directly measured and is always 0.10 ft.  $z_{GR}$  can be used to find  $I_{GR}$  in the following way.

$$I_{GR} = z_{GR} - 0.10 \text{ ft} = 1.284 \text{ ft}$$

These values for  $z_{GR}$  and  $I_{GR}$  will be used with all data for this storm event.

Next, in order to calculate hydraulic radius as described in Table 2, the flow depth  $d$  must be known. For now, assume  $d$  equals the current sensor reading average minus  $I_{GR}$ . After finding hydraulic radius,  $V$  and  $q$  can be calculated from Manning's equation as previously shown.

**Table 4.** Sample Green Roof sensor data.

Time	Sensor Reading (ft)
7:44	1.3639
7:45	1.3639
7:46	1.3640
7:47	1.3637
7:48	1.3634
7:49	1.3634
7:50	1.3628
7:51	1.3628
7:52	1.3624
7:53	1.3624
7:54	1.3633

For example, assume a sensor reading of 1.416 ft. Using the underdrain invert calculated above,

$$\overline{x_{GR}} = 1.416 \text{ ft}$$

$$d = x - I_{GR} = 0.13 \text{ ft}$$

$$d = 0.13 \text{ ft} < R_{GR}$$

$$c = d = 0.13 \text{ ft}$$

$$\varphi = 2 * \arccos\left(\frac{R_{GR} - c}{R_{GR}}\right) = 2.13 \text{ rad}$$

$$K = \frac{R_{GR}^2 * (\varphi - \sin(\varphi))}{2} = 0.041 \text{ ft}^2$$

$$s = R_{GR} * \varphi = 0.54 \text{ ft}$$

$$a = K = 0.041 \text{ ft}^2$$

$$P_w = s = 0.54 \text{ ft}$$

$$R_h = \frac{a}{P_w} = 0.076 \text{ ft}$$

$$V = \frac{1.49}{n} * R_h^{2/3} * \delta_{GR}^{1/2} = 0.42 \text{ ft/s}$$

$$q = V * a = 0.017 \text{ cfs} = 7.73 \text{ gpm}$$

It is important to note that the  $q$  calculated above corresponds to the total area under the water level, not the actual flow area. The next step is to find  $q_0$ , the flow corresponding to the zero-flow area shown in Figure 2. Manning's equation can be used to calculate  $q_0$  as well. The value of  $d$  for this calculation will be 0.1 ft, the height of the zero-flow depth above the underdrain invert.

$$d_0 = 0.10 \text{ ft} < R_{GR}$$

$$c_0 = d_0 = 0.10 \text{ ft}$$

$$\varphi_0 = 2 * \arccos\left(\frac{R_{GR} - c_0}{R_{GR}}\right) = 1.85 \text{ rad}$$

$$K_0 = \frac{r_N^2 * (\varphi_0 - \sin(\varphi_0))}{2} = 0.028 \text{ ft}^2$$

$$s_0 = R_{GR} * \varphi_0 = 0.46 \text{ ft}$$

$$a_0 = K_0 = 0.028 \text{ ft}^2$$

$$P_{w,0} = s = 0.46 \text{ ft}$$

$$R_{h,0} = \frac{a_0}{P_{w,0}} = 0.060 \text{ ft}$$

$$V_0 = \frac{1.49}{n} * R_{h,0}^{2/3} * \delta_{GR}^{1/2} = 0.36 \text{ ft/s}$$

$$q_0 = V_0 * a_0 = 0.010 \text{ cfs} = 4.51 \text{ gpm}$$

Because the value of  $d_0$  is constant, the value of  $q_0$  is likewise constant and does not need to be recalculated for new sensor readings or storm events.

The flow rate in the green roof underdrain is the difference between  $q$  and  $q_0$ . If  $q_0$  is larger than  $q$ , there is no flow.

$$q_{GR} = q - q_0 = 3.22 \text{ gpm}$$

### - V-Notch Weir Equation: Bioswale

The sensor at the bioswale is a pressure transducer which reads the depth of water just upstream of a V-notch weir (Figure 3). Flow rate over the weir is related to the head of water on the weir, the angle of the notch, and a weir coefficient through the following equation<sup>7</sup>:

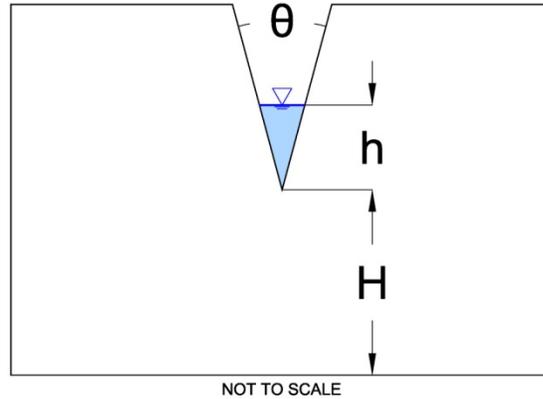
$$q = C_w * \tan\left(\frac{\theta}{2}\right) * h^{5/2}$$

$$\begin{array}{ll} q = \text{flow rate (cfs)} & \vartheta = \text{angle of notch} \\ C_w = \text{weir coefficient} & h = \text{weir head (ft)} \end{array}$$

<sup>7</sup> Source: Merritt, F.S., Loftin, M.K., and Ricketts, J.T. (1996). *Standard handbook for civil engineers*. McGraw-Hill, Inc., New York, NY, 21.64

$$H_{BS} = 0.504 \text{ ft}$$

$$\theta_{BS} = 30^\circ = 0.5236 \text{ rad}$$



**Figure 3.** Diagram of V-notch weir with important parameters illustrated.

The head on the weir is the difference between the sensor-read water level and weir depth. If the water level is below the weir depth, no flow is occurring. The coefficient  $C_w$  is dependent on both the weir head and angle of the notch, and for a 30° notch angle can be expressed in terms of head using the following equation<sup>8</sup>:

$$C_w = -0.3797 * h^3 + 1.1439 * h^2 - 1.1368 * h + 2.9251$$

For example, if the bioswale sensor reads a depth of 0.754 ft:

$$x_{BS} = 0.754 \text{ ft}$$

$$h = x_{BS} - H_{BS} = 0.250 \text{ ft}$$

$$C_w = -0.3797 * h^3 + 1.1439 * h^2 - 1.1368 * h + 2.9251 = 2.71$$

$$q = C_w * \tan\left(\frac{\theta_{BS}}{2}\right) * h^{5/2} = 0.048 \text{ cfs} = 10.2 \text{ gpm}$$

### - Orifice Equation: Main Site Outfall

Flow leaving the main site outfall is controlled by a 1.625-in (0.1354 ft) diameter<sup>9</sup> orifice, as shown in Figure 4. The flow rate at this location is based on measurements from two pressure transducers. The first (MO-ORI) is located immediately upstream of the orifice at the underground gravel detention bed; the second (MO-OUT) is at the main site outlet where runoff is discharged to the receiving stream. The difference between the heads read by the MO-

<sup>8</sup> Based on Fig. 21.61 from Merritt et al. 1996

<sup>9</sup> Urban Engineering & Associates, Inc. December 23, 2004. Plans for "Virginia Gateway Addition." Project Number 01-0437. Prince William County, V.A, Sheet 12A.

ORI sensor and the MO-OUT sensor is used to find the net pressure head that can be used to calculate flow through the orifice using the following equation<sup>10</sup>:

$$q = C_d * a_o * \sqrt{2 * g * \Delta h}$$

$a_o$  = flow area through orifice (ft<sup>2</sup>)       $q$  = flow rate (cfs)  
 $g$  = gravitational acceleration (ft/s<sup>2</sup>)       $C_d$  = discharge coefficient  
 $\Delta h$  = difference in head (ft)

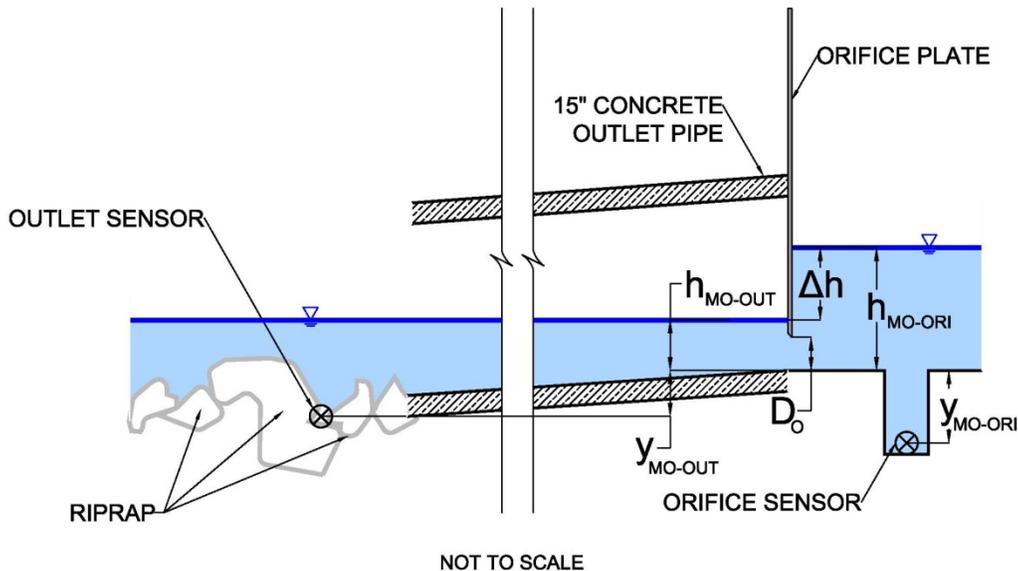
$$y_{MO-ORI} = 0.66 \text{ ft} \quad y_{MO-OUT} = 0.63 \text{ ft}$$

$$D_o = 0.1354 \text{ ft} \quad g = 32.2 \frac{\text{ft}}{\text{s}^2}$$

If the water level is below the orifice invert on either side of the orifice, the head on that side is zero. For example, if the detention bed sensor reads a value of 0.61 ft

$$x_{MO-ORI} = 0.61 \text{ ft} < y_{MO-ORI}$$

$$h_{MO-ORI} = 0 \text{ ft}$$



**Figure 4.** Diagram of orifice at Main Site Outfall, with important parameters illustrated.

If the water level is above the orifice invert, the head is calculated by subtracting the distance from the invert to the sensor from the sensor reading. If the detention bed sensor reads a value of 0.75 ft,

$$x_{MO-ORI} = 0.75 \text{ ft} > y_{MO-ORI}$$

$$h_{MO-ORI} = x_{MO-ORI} - y_{MO-ORI} = 0.09 \text{ ft}$$

<sup>10</sup> Source: Lindeburg, M.R. (2008). *Civil engineering reference manual for the PE exam*. Professional Publications, Inc., Belmont, CA, 17-19

As long as at least one side has head, flow through the orifice will occur from the side with greater head to the side with less head. The net head is the difference between the heads on the two sides, and is calculated as follows.

$$\Delta h = |h_{MO-ORI} - h_{MO-OUT}|$$

The coefficient of discharge is not constant, and is dependent on the net head across the orifice. The following relationship<sup>11</sup> is used to calculate a coefficient of discharge using net head for a 0.1354-ft diameter, circular, sharp-edged orifice:

$$C_d = 0.00021(\Delta h)^4 - 0.00259(\Delta h)^3 + 0.01172(\Delta h)^2 - 0.232(\Delta h) + 0.61816$$

Finally, the submerged area of the orifice needs to be determined. Submerged area is determined based on the side with greater head. If that head is greater than the orifice diameter, then the orifice is fully submerged, and the area is as follows.

$$a_o = \frac{1}{4} * \pi * (D_o)^2 = 0.0144 \text{ ft}^2$$

If the orifice is only partially submerged, the area can be found using Steps 1-3 and Step 5 from Table 2. For example, to find the submerged area for  $h_{MO-DET} = 0.09 \text{ ft}$

$$d = h_{MO-ORI} = 0.09 \text{ ft} > \frac{D_o}{2}$$

$$c = D_o - d = 0.0454 \text{ ft}$$

$$\varphi = 2 * \arccos\left(\frac{\frac{D_o}{2} - c}{\frac{D_o}{2}}\right) = 2.47 \text{ rad}$$

$$K = \frac{(\frac{D_o}{2})^2 * (\varphi - \sin(\varphi))}{2} = 0.0042 \text{ ft}^2$$

$$a_o = \frac{1}{4} * \pi * (D_o)^2 - K = 0.0102 \text{ ft}^2$$

Assume a scenario in which the detention bed sensor reads 0.75 ft, and the stream sensor reads 0.70 ft. The flow through the orifice would be calculated in the following way.

$$x_{MO-ORI} = 0.75 \text{ ft} > y_{MO-ORI}$$

$$h_{MO-ORI} = x_{MO-ORI} - y_{MO-ORI} = 0.09 \text{ ft}$$

$$x_{MO-OUT} = 0.70 \text{ ft} > y_{MO-OUT}$$

$$h_{MO-OUT} = x_{MO-OUT} - y_{MO-OUT} = 0.07 \text{ ft}$$

$$\Delta h = |h_{MO-ORI} - h_{MO-OUT}| = 0.02 \text{ ft}$$

<sup>11</sup> Based on Table 21.9 from Merritt et al. 1996

$$d = h_{MO-ORI} = 0.09 \text{ ft} > \frac{D_o}{2}$$

$$c = D_o - d = 0.0454 \text{ ft}$$

$$\varphi = 2 * \arccos\left(\frac{\frac{D_o}{2} - c}{\frac{D_o}{2}}\right) = 2.47 \text{ rad}$$

$$K = \frac{\left(\frac{D_o}{2}\right)^2 * (\varphi - \sin(\varphi))}{2} = 0.0042 \text{ ft}^2$$

$$C_d = 0.00021(\Delta h)^4 - 0.00259(\Delta h)^3 + 0.01172(\Delta h)^2 - 0.232(\Delta h) + 0.61816 = 0.614$$

$$a_o = \frac{1}{4} * \pi * (D_o)^2 - K = 0.0102 \text{ ft}^2$$

$$q = C_d * a_o * \sqrt{2 * g * \Delta h} = 0.00718 \text{ cfs} = 3.19 \text{ gpm}$$

All flow from the detention bed to the stream is reported as a positive value, and all flow from the stream to the detention bed ("backflow") is reported as a negative value. This distinction is very important, as it allows the program to exclude backflow when finding runoff volume and peak runoff rates in the runoff analysis section.

# Runoff Analysis

Each calculation performed during runoff analysis is described and demonstrated below. If any simplifications were necessary to automate the calculation, those are described as well.

## - Runoff Volume

The sensors all record a data point every minute, allowing a flow rate to be calculated each minute during a storm event. Plotting these calculated flow rates against time gives the hydrograph for that monitoring station during the storm event. The total runoff volume is the area underneath that hydrograph, which is calculated using a Riemann sum as shown in the example below.

Assume that runoff for the storm shown in Table 5 began at 2:00 and ended at 20:00. The Riemann sum calculation of runoff volume is:

$$Q = (1 \text{ min}) * (0 \text{ gpm} + 1 \text{ gpm} + 5 \text{ gpm} + \dots + 1 \text{ gpm} + 0 \text{ gpm})$$

This calculation can be simplified by distributing the 1-minute rectangle thickness:

$$Q = 0 \text{ gal} + 1 \text{ gal} + 5 \text{ gal} + \dots + 1 \text{ gal} + 0 \text{ gal}$$

Because all sensors use the 1-minute data collection interval, the sum of flow rates in gallons per minute is equal to the total runoff volume in gallons.

### *\*\*Simplification: Hydrograph tail termination*

As runoff slowly decreases following a storm, the depth of flow approaches the minimum accuracy of the sensors. To prevent reporting data with limited accuracy for up to several days following a storm event, the flow rates are automatically set to zero by the program as the hydrograph tails become approximately horizontal. This improves the ability of the program to display accurate data and show important hydrograph features, but may result in slight underestimation of total runoff volume for some storms.

**Table 5.** Sample runoff data for Riemann sum example calculation.

Time	Flow rate (gpm)
2:00	0
2:01	1
2:02	5
...	...
19:59	1
20:00	0

The hydrograph tail is terminated at the first point for which both of the following criteria are met. The “change in depth” referred to below is the difference between the current sensor value and the value recorded by that sensor 1 minute ago.

- The average change in depth for all remaining values in the dataset is negligible. – Sensor readings, taken in inches, are recorded to five decimal places (0.00001 in). However, readings beyond the third decimal place (0.001 in) are subject to drift and not considered significant. Therefore, if the average of all remaining changes is on the order of a thousandth of an inch, the total remaining change in depth is assumed to be negligible.
- No sudden changes in depth remain in the dataset. – Sensor readings varying by more than one tenth of an inch (0.1 in) are considered sudden and well outside the expected normal sensor drift. The tail of the hydrograph cannot be truncated prior to the appearance of changes of this magnitude.

### - Peak Runoff Rate

No calculation is required beyond the flow rate calculations described in the previous section; the maximum flow rate calculated is the peak runoff rate for that monitoring station.

### - Runoff Retention

Runoff retention is the fraction of rainfall that is not discharged from the site as runoff. For example, if a storm produces 5,000 gal of rainfall but only 800 gal of runoff are discharged from the site, then:

$$r = \frac{5000 \text{ gal} - 800 \text{ gal}}{5000 \text{ gal}} = 84\%$$

### - Time of Concentration

One of the many methods to calculate time of concentration of a watershed is to relate it to the lag time (time to peak) using this equation<sup>12</sup>:

$$t_c = \frac{5}{3} * t_L$$

The lag time for a watershed has several definitions. For this application, lag time is calculated as the time from when the centroid value of rainfall occurs to when the peak runoff rate value occurs. This definition is based on the SCS triangular hydrograph approximation shown in Figure 5.

For example, assume a storm in which rainfall began at 12:00 and fell at a constant rate until the storm ended at 16:00. The peak runoff rate for this storm was observed to occur at 17:00. In this case, because the rainfall rate was constant, the rainfall centroid occurred exactly halfway through the storm at 14:00. The lag time is the time between the centroid (at 14:00) and the peak runoff rate (at 17:00), which is 3 hr. Time of concentration is then calculated like this:

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<sup>12</sup> Source: Maidment, D.R. (1993). *Handbook of hydrology*. McGraw-Hill, Inc., New York, NY, 9.23 to 9.25.

$$t_c = \frac{5}{3} * t_L = 5 \text{ hr}$$

**\*\*Simplification: Rainfall segments**

The time of concentration calculation is never as simple as the triangular hydrograph approximation suggests. Very seldom do actual rainfall events have a constant intensity, and many have breaks of several hours where no precipitation occurs at all. In storms with multiple discrete rainfall segments, there are often multiple runoff peaks that each correspond to a different segment of the storm. In order to correct for this effect during segmented storms, the time of concentration calculation is based on data from the first rainfall segment rather than the total storm event.

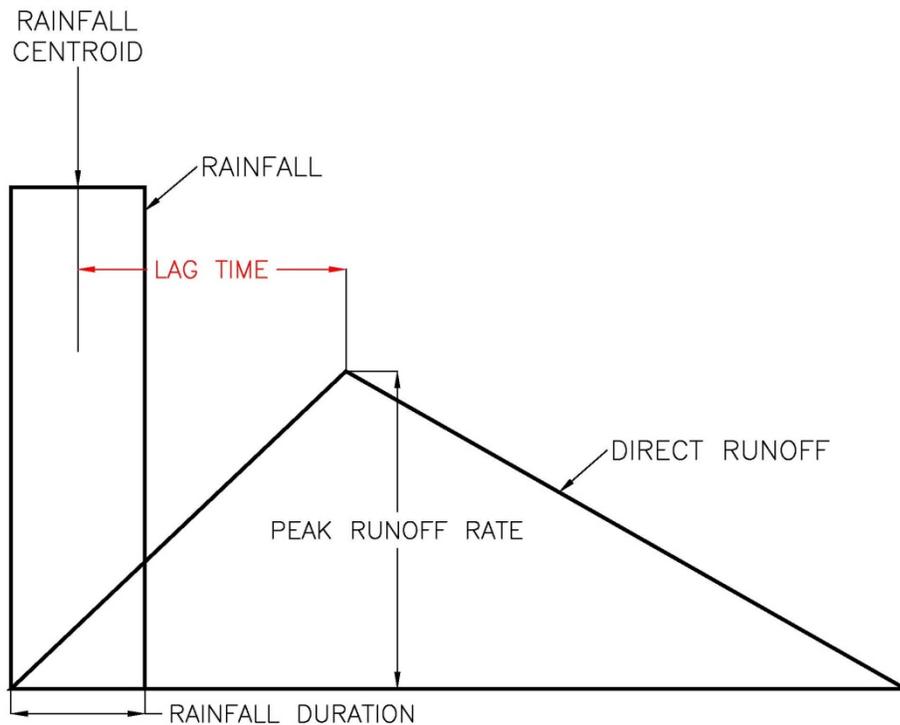
**- Curve Number**

The curve number for the site is calculated using the total volume of rainfall,  $P$ , and runoff,  $Q$ , for each storm event. The NRCS curve number equation<sup>13</sup> is:

$$Q = \frac{(P - I_a)^2}{P - I_a + S}$$

In the equation above  $S$  is the potential retention and  $I_a$  is initial abstractions.  $I_a$  is commonly approximated as:

$$I_a = 0.2 * S$$



**Figure 5. SCS triangular hydrograph diagram with lag time highlighted.**

<sup>13</sup> Source: Lindeburg, M.R. (2008). *Civil engineering reference manual for the PE exam*. Professional Publications, Inc., Belmont, CA, 20-15 to 20-19

Using this approximation to eliminate the initial abstractions term, the curve number equation can be rewritten as:

$$Q = \frac{(P - 0.2 * S)^2}{P + 0.8 * S}$$

The equation can then be rewritten again as a quadratic:

$$0 = \alpha_1 * S^2 + \alpha_2 * S + \alpha_3 = 0.04 * S^2 + (-0.4 * P - 0.8 * Q) * S + (P^2 - P * Q)$$

$$\alpha_1 = 0.04$$

$$\alpha_2 = -0.4 * P - 0.8 * Q$$

$$\alpha_3 = P^2 - P * Q$$

The coefficients  $a$ ,  $b$ , and  $c$  can be used to solve for  $S$  via the quadratic formula as shown here:

$$S = \frac{-\alpha_2 - \sqrt{\alpha_2^2 - 4 * \alpha_1 * \alpha_3}}{2 * \alpha_1}$$

Notice that, instead of the “ $\pm$ ” operation which would normally give two solutions to the quadratic formula, in this case only the “ $-$ ” is used because there is only one valid curve number for the site. Finally, the following equation relates  $S$  and the curve number  $CN$ .

$$CN = \frac{1000}{S + 10}$$

For example, assume a storm dropped 0.80 in of precipitation on a subcatchment, and this resulted in 0.12 in of runoff. The steps required to calculate a curve number in this situation are as follows.

$$\alpha_1 = 0.04$$

$$\alpha_2 = -0.4 * P - 0.8 * Q = -0.416$$

$$\alpha_3 = P^2 - P * Q = 0.544$$

$$S = \frac{-\alpha_2 - \sqrt{\alpha_2^2 - 4 * \alpha_1 * \alpha_3}}{2 * \alpha_1} = 1.53 \text{ in}$$

$$CN = \frac{1000}{S + 10} = 87$$

### - Rational Method Coefficient

The rational method equation relates watershed area in acres,  $A$ , and peak rainfall intensity in in/hr,  $i_p$ , to a storm event’s peak runoff rate in cfs,  $q_p$ . The equation<sup>14</sup> is shown below:

$$q_p = C * i_p * A$$

<sup>14</sup>Source: Lindeburg, M.R. (2008). *Civil engineering reference manual for the PE exam*. Professional Publications, Inc., Belmont, CA, 20-13

The WSSI site is made up of four separate subcatchments (the West Outfall, Main Site Outfall, Bioswale, and Northeast Outfall). The total watershed area can be calculated by taking the sum of the subcatchment areas<sup>15</sup>.

$$A_{WO} = 28,500 \text{ ft}^2 = 0.65 \text{ ac}$$

$$A_{MO} = 89,900 \text{ ft}^2 = 1.97 \text{ ac}$$

$$A_{BS} = 25,800 \text{ ft}^2 = 0.59 \text{ ac}$$

$$A_{NE} = 4,400 \text{ ft}^2 = 0.10 \text{ ac}$$

$$A_{total} = A_{WO} + A_{MO} + A_{BS} + A_{NE} = 3.31 \text{ ac}$$

The rational method coefficient,  $C$ , can be calculated as demonstrated in the following example.

Assume a storm had a peak rainfall intensity of 1.0 in/hr and the peak runoff rate from the WSSI site was 0.8 cfs. The coefficient would be calculated like this:

$$C = \frac{q_p}{i_a * A_{total}} = 0.24$$

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<sup>15</sup>Source: Urban Engineering & Associates, Inc. December 23, 2004. Plans for "Virginia Gateway Addition." Project Number 01-0437. Prince William County, V.A, Sheet 12.

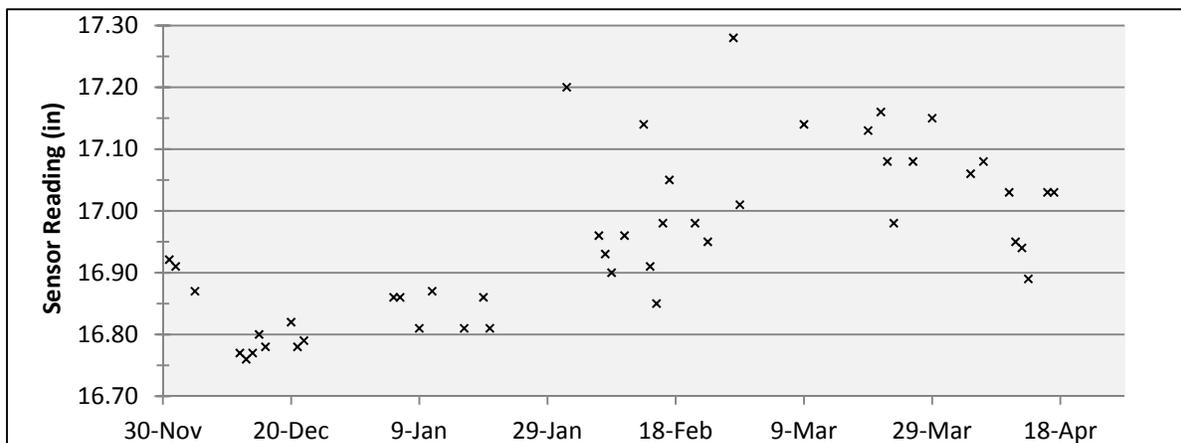
## Appendix A – Discussion of Error

Many potential inaccuracies exist within the data collection and analysis methods outlined in this report. It is important to consider these sources of error when interpreting the results of analysis.

### - Drifting Sensor Values

The most significant source of error within the system is that the values reported by the sensors seem to “drift” over time. The most obvious example of this is zero-flow depth (Figure A- 1), which never actually changes in elevation but tends to be reported differently over time by the same sensor. The variation in sensor values can be as large as several tenths of an inch, which can dramatically alter the calculated flow rates. The calculations at the Main Site Outfall, which are based on the readings of two pressure transducers rather than just one, are the most susceptible to error caused by sensor drift. In order to protect the accuracy of zero-flow depth values from corruption due to sensor drift, all values are calculated on a per-storm basis using sensor values immediately preceding the storm. Thus far, no discernible pattern or cause for the drifting values has been identified.

While the cause of sensor drift may be unknown, it can easily be recognized by looking at the raw data. If the values reported by the sensor immediately following the end of runoff from a storm event are dramatically different than the zero-flow depth, it is likely that sensor drift has impaired the accuracy of data for that storm. Although sensor drift creates many problems for analysis, some information can still be gleaned from the data even if sensor drift has occurred. For example, the shape of the hydrograph and time of peak runoff rate should be unaffected.



**Figure A- 1.** West Outfall sensor readings for zero-flow depth plotted over time. These readings should be constant since they all correspond to the same "real-world" elevation, but instead show a high degree of variability due to sensor drift.

### - Time of Concentration

Like zero-flow depth, time of concentration should be a constant value for each subcatchment. However, these values demonstrate a high degree of variability, attributable to the varied nature of storm events and the limitations of an idealized SCS hydrograph. Rather than report a time of concentration for each storm event, only the average time of concentration for all analyzed storms is provided by the website. It is expected that this long-

term average will minimize the influence of individual storms and be more descriptive of subcatchment characteristics.

For some storms, the time of concentration values calculated by the program are either unrealistically large (e.g. 4 hours for a 0.1 ac subcatchment) or negative. In cases such as these it is assumed the storm's characteristics were inappropriate for the calculations methods used, and the results are excluded from the long-term average to preserve its accuracy.

### **- Weir and Orifice Coefficients**

The report describes polynomial equations with head as the dependent variable for both the weir and orifice coefficients. These equations carry no physical meaning, and are merely expressions fit to the values read from figures and tables in the resources. Despite their empirical basis, both equations give very accurate results over the range of heads used to generate them. The most significant concern surrounding their use is extrapolation to heads beyond the ranges provided in the resources. This situation would only be observed in very low-frequency storms, such as a 100-yr event. For all other storms, the weir and orifice coefficients are expected to be a negligible source of error.

### **- Freezing Temperatures**

The sensors used to monitor water level were designed to be used for liquid water and not snow or ice. When water freezes in the stilling wells it can lead to bizarre sensor readings, most commonly in the form of impossibly large depth values. Fortunately, due to Virginia's temperate climate this is typically only an issue during late fall and winter. When analyzing a storm event between November and March, it is recommended to verify that the temperatures were above freezing for the full duration of the event.

## Appendix B: Parameters Determined Through Experimentation

**Table B- 1.** Description of methods used to experimentally determine important parameters

Parameter being determined	Known parameter(s)	Necessary equation(s)	Brief description of procedure
effective pipe slope, $\delta$	Manning's roughness coefficient, $n$ ; sensor reading, $x$ ; pipe radius $R$ ; flow rate, $q$	$\delta = \left( \frac{qn}{1.49R_h^{2/3}} \right)^2$	Flow over each stilling well was generated either by a storm or with a hose. The flow rate was measured manually (using a 5-gal bucket and stopwatch), and sensor readings were used to calculate the hydraulic radius (see Table 2). Manning's equation was rearranged to isolate and solve for slope.
green roof calibration coefficient, $C_{GR}$	sensor reading, $x$	none	A hose was used to generate flow over the green roof stilling well. The stilling well was observed and the exact time when flow began was noted. The sensor reading corresponding to the beginning of flow (i.e. $q$ just slightly larger than 0 gpm) was 0.021 ft higher than the stagnant water depth.